Rocket Thrust

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1 Objective

This report describes the procedure used to measure the thrust of a rocket motor. The rocket motor is to be mounted on an aluminum beam which will deflect proportionally to the thrust of the rocket. The minute deflections of the beam will be measured with a strain gauge mounted to the beam and the strain gage will supply a voltage to a filter, an amplifier, and ultimately to a computer which will rapidly record the values of the strain. Sources of error, such as may exist in this experiment will be tabulated to give the confidence to which the results may be stated.

2 Theory

This section proposes to discuss the theory required for force measurement.

2.1 Beam Theory and Sensor Bandwidth

2.1.1 Beam Theory

The rocket will be mounted on the extreme end of a beam, and the opposite end of the beam will be held fixed. The beam will flex under the force of the rocket as the rocket provides a moment about the fixed end of the beam. This moment tends to compress the bottom surface of the beam and stretch the top surface[1].

How much the beam deflects for a given rocket thrust depends on its "elastic modulus," a measure for the stretchiness of the beam, the length of the beam, and a geometrical feature of the beam known as the "second moment of area." The elastic modulus is a material property, well known for various types of metals. In our case, the elastic modulus of aluminum is known at 70 GPa, and linearly relates the stretch in the beam to the applied pressure, known as "stress."

[1]Image courtasy of NI.com. See at http://www.ni.com/cms/images/devzone/tut/half_bridge_2.png

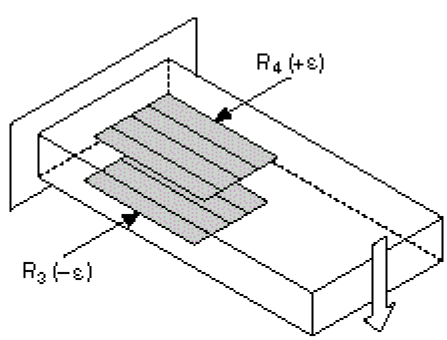

Figure 1: A beam in bending with a half bridge.

$$I = \frac{b * h^3}{12}$$

where b is the width of the beam, and h is its thickness. There are two measures for the length of the beam, one measuring from the fixed support to the center of the motor's thrust, and another measuring from the center of the strain gauge to the center of the motor's thrust.

The beam's greatest deflection can be calculated as

$$\delta = \frac{PL^3}{3EI}$$

Where δ is the deflection, P is the force of the rocket, E the elastic modulus, and I is the second moment of area mentioned above. L in this case is the distance from the center of the rocket to the fixed support.

The stretch or compression of the metal at the surface of the beam is given by

$$\frac{6PL'}{Ebh^2}$$

with L' the length of the beam from the center of the rocket to the center of the strain gauge.

2.1.2 Bandwidth

The beam has several limitations to it. First of all, the beam must not yield from the force of the rocket. If it does, the applied force is no longer linearly proportional to the strain in the beam. However, the beam must deflect sufficiently that there is an appreciable change in the length of the surface material of the beam. We selected $1000\mu\epsilon$ or micro-strain for a measurable change in length. This corresponds to a 0.1% change in the length of the surface of the beam.

Now a beam with an applied force and no damping will vibrate like a pitch fork with a natural frequency. If the rocket's variation in thrust corresponds to the natural frequency of the beam, the beam will vibrate back and forth at amplitudes much higher than the deflection due to the force of the rocket. The natural frequency, ω_n, of a beam's vibration is given by the equation

$$\omega_n = \sqrt{\frac{k}{m}}$$

where k is the spring constant of the beam and m is the equivalent mass at the end of the beam. The spring constant is

$$k = \frac{bh^3E}{4L'^3}$$

and the equivalent mass is given by

$$M_{eq} = M_{rocket} + \frac{1}{4}M_{beam}$$

2.2 Strain Gauge Theory

A strain gauge is a piece of wire, usually copper wire, with a resistance dependant on the length of the wire. In general the resistance of an object, R, is

$$R = \frac{\rho * L}{A}$$

where ρ is the resistivity of the material, L is the length between the anode and cathode, and A is the cross sectional area. Now as a wire stretches, its length increases and its cross sectional area decreases according the the Poisson's ratio, another material property. An ideal strain gauge, therefore is long piece of wire affixed to a point on an object that strains equally to the object. As the object strains, the resistance of the strain gauge increases as well. In general, the strain in a material is at most tenths of a percent and in our case is less than 1 tenth of one percent. This corresponds to a similarly tiny change in resistance of the resistor.

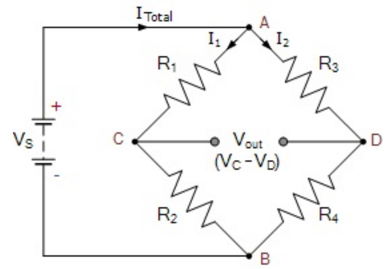

Figure 2: The Wheatstone Bridge

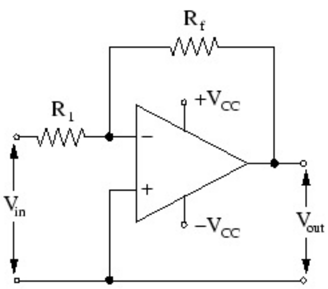

Figure 3: A simple amplifying circuit

2.3 The Wheatstone Bridge

A Wheatstone bridge[2] is an arrangement of resistors that allow small changes in the resistance of one or two resistors to accompany large relative changes in voltage. Simply put a voltage is applied between A and B. If the resistances of the four resistors is the same, then the voltage difference between C and D remains 0. However, if the resistance of R_1 decreases, then the voltage at C will exceed the voltage at D. Using this principal, the wheatstone bridge allows up to 4 strain gauges to be attached to a beam and a voltage will be delivered if the resistances don't match.

The net output voltage can be calculated using the equation

$$\frac{\Delta E_{out}}{E_{in}} = \frac{1}{4}F[\epsilon_1 - \epsilon_2 - \epsilon_3 + \epsilon_4]$$

where F is a gauge factor that depends on how many strain gauges are used, E refers to a voltage difference, and ϵ refers to the strain in a particular strain gauge.

2.4 Signal Conditioning

The raw data output by the strain gauges may include "noise" not due to the thrust of the rocket. It is also very likely to be insufficiently strong for the computer to pick up and record. For that reason, the signal is conditioned through two components: a filter and an amplifier.

The amplifier uses a simple operational amplifier and increases the voltage by a gain.[3] In this case, the amplifier increases the input voltage according to

$$\frac{V_{out}}{V_{in}} = -\frac{R_f}{R_i}$$

The filter is a type known as a "low pass" filter, meaning that it filters out changes in signal that happen at high frequencies. Much like the walls of an apartment that filter out the melody of the neighbor's loud

[2] image courtesy of Electronics Tutorials http://www.electronics-tutorials.ws/articles/wheatstone1.gif?x98918

[3] This image can be had at http://webpages.ursinus.edu/lriley/ref/circuits/node5.html along with an excellent discussion of operational amplifiers.

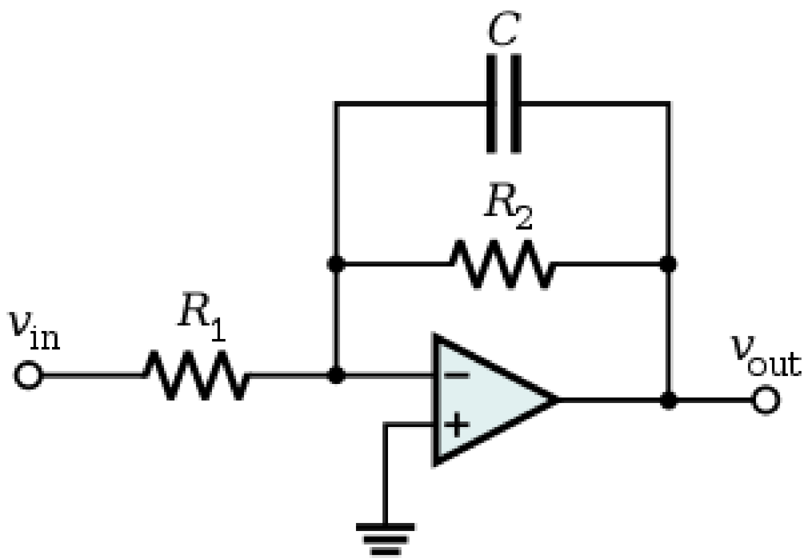

Figure 4: An active low-pass filter

music but not the low frequency beat. In our case, however, the filter acts on electrical signals instead of on auditory signals. A schematic[4] is below.

This filter has a cutoff frequency of

$$f_c = \frac{1}{2\pi R_2 C}$$

which means that electrical signals occurring at frequencies above the cutoff frequency are will distribute less than half the power of the input.

When the amplifier and filter are chained together, the result is a circuit that removes unwanted information and increases desirable information, both for output to the computer.

3 Procedure

The procedure consists of a beam specifically to measure a specific force. After the sensor is designed, it must be built, have the strain gauges mounted, create circuits, and then test the motors.

3.1 Characterization of Forces

Standard rocket thrust data is given on the Estes website.[5] The data is presented below.

We find from this data that the maximum force provided by the rocket is some 14 newtons, and that this force is provided in a nearly linearly increasing line up to 14 newtons. The initial impulse of the rocket occurs over the course of 0.2 seconds. After the initial burst of thrust the total thrust settles to 4 or 4.5 newtons over another 1.6 seconds.

3.2 Design Sensor

In order to make the beam function for measuring the thrust from the rocket, it must flex an appreciable amount without yielding and it must have a high enough natural frequency that the beam system reacts at zero phase relative to the impulse of the rocket motor.

[4]courtasy of the Wikimedia foundation. Please donate at https://wikimediafoundation.org/wiki/Ways_to_Give or visit the image at https://upload.wikimedia.org/wikipedia/commons/thumb/5/59/Active_Lowpass_Filter_RC.svg/300px-Active_Lowpass_Filter_RC.svg.png

[5]Which can be found here http://www.estesrockets.com/001616-c6-0

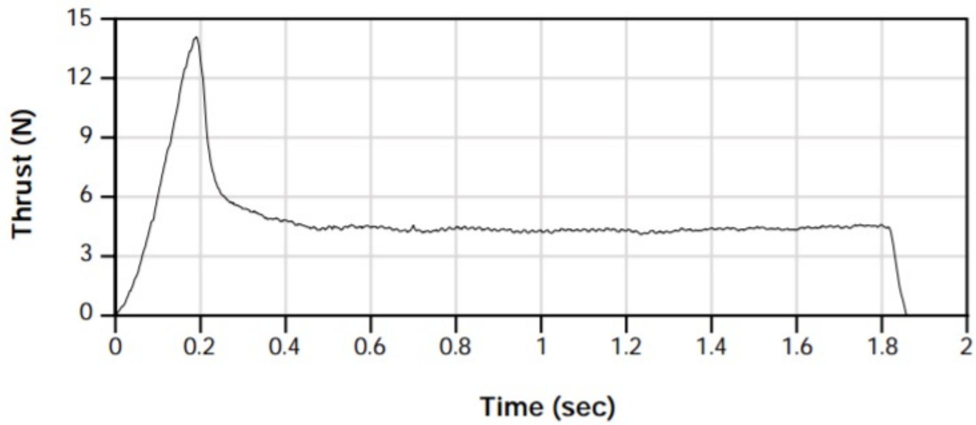

Figure 5: The thrust data from the Estes website of their C-6 rocket engine

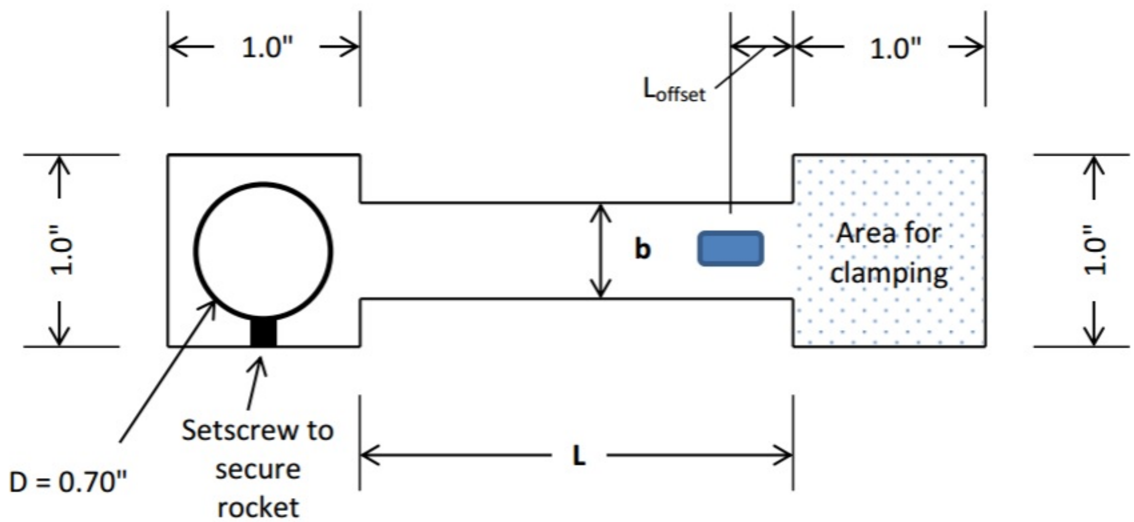

Figure 6: A schematic of the beam, top view. At the left is the hole through which the rocket is affixed and at the right is the aread where the beam is clamped onto a fixed, horizontal surface.

Unlike a traditional scale, the force sensor will be designed specifically to measure the thrust of the C-6 Estes rocket engine. As such, it was designed specifically to match the engine.

Figure 6 is a top view of the beam. The round rocket engine fits in the circle labeled "D = 0.70" " and is secured with a set screw. In the dimension perpendicular to the screen is selectable from available thicknesses of aluminum bar. Aluminum bar is available in 1/16", 1/8", and 3/16". The length of the beam and width of the beam are then modulated to provide the necessary flex with the given approximate force of the rocket ($1000\mu\epsilon$). The natural frequency of the beam was specified at a value greater than 80 Hz.

Using the equations described in the Beam Theory section, we found that a the dimensions of

$$L = 3.543\text{"}$$

$$b = 0.433\text{"}$$

$$h = 1/8\text{"}$$

3.3 Machine Sensor

The sensor was machined as drawn, of aluminum, but with the minor modification that fillets were inserted at all internal corners.

3.4 Mount and Wire Strain Gages

For simplicity and reliability, the beam operates on a "half bridge" circuit, with one strain gauge on the top surface and another strain gauge on the bottom surface.

The two strain gauges are wired into positions 1 and 2 as described in the "Wheatstone Bridge" section. (See Figure2), and are mounted firmly to the surface of the beam. It's important that the strain gauge be mounted with an adhesive that does not strain in shear easily so that the strain in the strain gauge matches the maximum strain at the surface of the beam. Since the adhesive is very thin, it may be thicker than the layer of oils and dirt that may be on the surface of the beam. To adhere the strain gauge directly to the beam, be sure to abrade and clean the beam thoroughly.

3.5 Amplify and Condition Signal

The circuits were designed to filter out high frequencies and to amplify to amplify significantly. As laid out in Figures 3 and 4, the values of the components are as follows:

$$R_l = 4.7k\Omega$$

$$R_f = 100\Omega$$

for the amplifier and

$$R_1 = R_2 = 4.7k\Omega$$

$$C = 0.33\mu F$$

for the low pass filter. This gives an amplification of 47 times and filters out frequencies above approximately 103 Hz.

3.6 Calibrate Sensor

After the sensor is built, it will be calibrated by applying weights. By fitting a line to the output values, an equation such as

$$force = sensitivity * voltage + offset$$

will allow the operators to determine the force exerted on the end of the beam. The initial calibration of the beam yielded the equation:

$$force = 5.503 * V_{output} - 0.750$$

3.7 Measure Rocket Engine Thrust

The rocket engine will then be tested for thrust.

4 Summary of Results

4.1 Calibration

On the day of the test, the transducer was calibrated for force with the following results: Note that each step in the Figure 7 corresponds with a different weight, loaded onto it, but that the weights were not loaded in order. Instead, the beam was loaded as follows: 0 grams, 500 grams, 1000 grams, 2000 grams, 1500 grams. Each of these loadings has a mean and standard deviation as listed in the table:

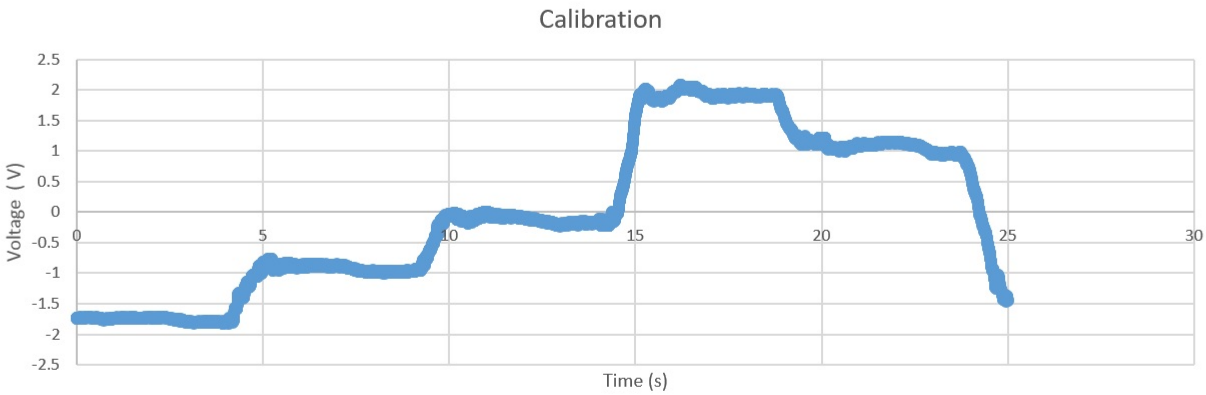

Figure 7: The calibration of the beam. Masses were loaded onto the beam in the following order: 0g, 500g, 1000g, 2000g, 1500g, 0g.

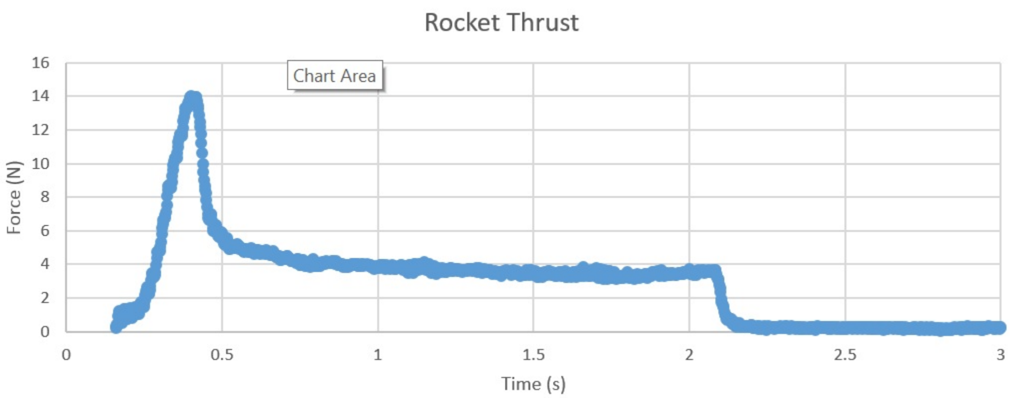

Figure 8: The thrust of the rocket in newtons. The time value occurs at an arbitrary point, and therefore does not align exactly with the data in Figure 5.

Mass	Mean Voltage	Std. Dev
0g	-1.7396	0.04319
500g	-0.9004	0.039419
1000g	-0.10151	0.051204
1500g	1.08946	0.048113
2000g	1.910403	0.016403

Which averages may be used to form a regression line, whose equation transforms voltage into mass and is:

$$mass = 5.2554 * V_{output} + 9.5384$$

and which has an r^2 value of 0.9954. The equation may then be used to calculate the force based on the voltage output of the rocket.

4.2 Rocket Thrust curves

The rocket thrust curves are displayed in Figure 8 and should be compared to the data presented in Figure 5.

5 Discussion

5.1 Comparison to Theory

As will be discussed in the subsection called "Error Propagation," there are many sources of known error in the theory section. There are also sources of error that are unknown. The knowable errors and unknowable errors compound and may result in dramatic differences between the theoretical values and the experimental values. In order to prevent compounded errors from delivering totally inaccurate results, the force transducer is calibrated prior to use. The calibration step relies on only two assumptions: first that the behavior of the transducer will be identical while loaded in the calibration routine as it will in the test routine.

5.2 Error Propagation

5.2.1 Sources of Uncertainty

In order to discuss error propagation, let's begin with a partial list of uncertainty sources that affect the difference between the true force acting on the beam and the number recorded by beam.

1. Beam length
2. Beam width
3. Beam thickness
4. Beam mass
5. Beam equivalent mass
6. Beam density variation (assumed 0)
7. Beam elastic modulus
8. Beam elastic modulus temperature dependence (assumed 0)
9. Beam mounting (assumed perfectly fixed)
10. Beam angle (assumed 0)
11. Rocket angle relative to beam (assumed perpendicular)
12. Strain gauge resistance
13. Strain gauge Gauge Factor
14. Strain gauge elastic modulus (assumed 0)
15. Strain gauge creep (assumed 0)
16. Temperature difference between strain gauges
17. Resistance in amplifier circuit
18. Resistance in filter circuit
19. Capacitance in filter circuit
20. Operational Amplifier current draw (assumed 0)
21. Resistance in leads (assumed 0)
22. Data Acquisition Unit error (assumed 0)

A calibration routine does not have as many sources of error. In fact, the sources of error from the above list that remain are only the following:

1. Beam elastic modulus temperature dependence
2. Strain gauge creep (assumed linearly elastic)
3. Rocket angle relative to calibration angle (the error corresponding to the angle error between rocket and beam)
4. Temperature difference between strain gauges

The calibration routine introduces some sources of error itself such as:

1. Position of calibration weights on beam
2. Vibration of calibration weights
3. True masses of calibration weights
4. Behavior differences under calibration routine compared to test routine

We may see that there are far fewer sources of error by using the calibration routine compared to the theoretical error calculation.

5.2.2 Uncertainty Relative Importance

Error sources have varying relative importance, which may be understood as follows. The values that determine the behavior of the transducer all are linear. In short they multiply to make a single factor, the "sensitivity" of the transducer to form the following equation.

$$force = sensitivity * voltage$$

In order to form that equation, all of the components of the transducer are multiplied together at varying degrees. They may be multiplied. Many are first degree having powers[6] of 1 or -1, but some have degrees of 2 or $\frac{1}{2}$ or even 3.

Each source of error is assumed independent of the others, and they sum as vectors do. Here, each value U is a source of uncertainty.

$$U_{total} = \sqrt{U_1^2 + U_2^2 + U_3^2 + \cdots}$$

For higher degree factors, the uncertainty is given by

$$U_{total} = \sqrt{D_{U_1} * U_1^2 + D_{U_2} * U_2^2 + D_{U_3} * U_3^2 + \cdots}$$

Where D_{U_n} indicates the degree that that factor takes in the overall factor.

5.3 Thrust Data

The manufacturers thrust data is given in Figure 5 and our test data is given in 8. Both figures reveal a peak thrust of approximately 14 newtons in a peak lasting approximately 0.15 seconds. They also both reveal a continuous thrust following the initial peak of 1.6 seconds.

5.3.1 Peak Thrust

The peak thrust is presented in Figure 9, which reveals several phenomena of note. First, there is a periodicity approximately 50Hz, and the total thrust of the rocket is achieved for only a few hundredths of a second.

5.3.2 Time Delay Until Ejection Charge

In order to launch the parachute for the model rocket, the engine has an ejection charge. This is a sudden burst of thrust in the opposite direction of the rocket's main thrust. The impulse is so brief that the strain gauge is set to vibrating and presenting a step change in junk data as presented in figure 10. The delay until the ejection charge occurs at approximately 6.5 seconds, or 4.3 seconds after the main thust has ceased.

[6] In this analysis, the absolute value of the degree of the factor is all that matters.

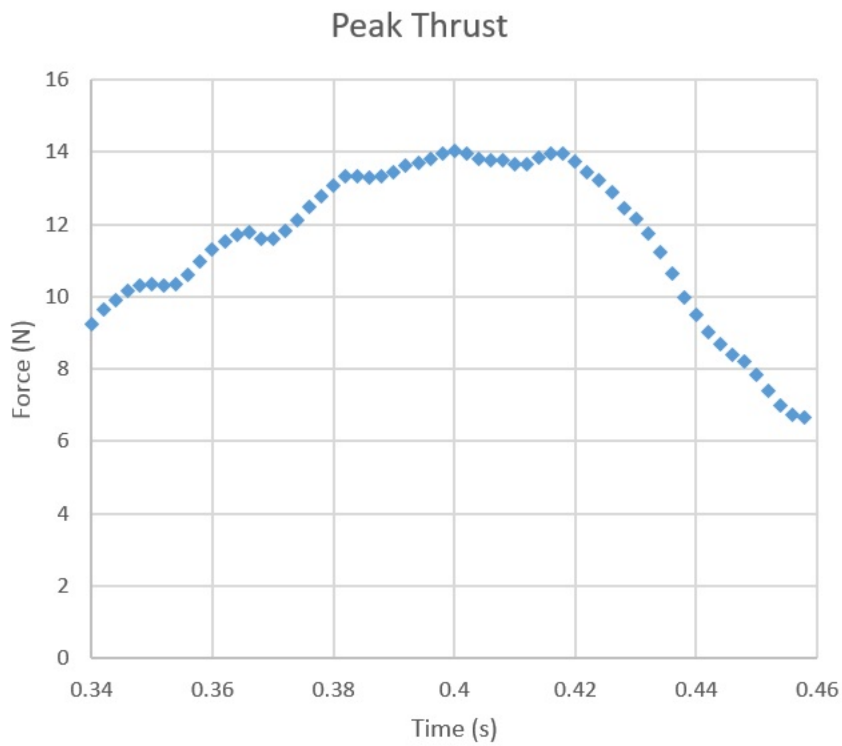

Figure 9: This is the peak thrust of the rocket. The rocket exceeds 10 newtons for only $8/100^{ths}$ of a second, and achieves its peak thrust for only about $2/100^{ths}$ of a second.

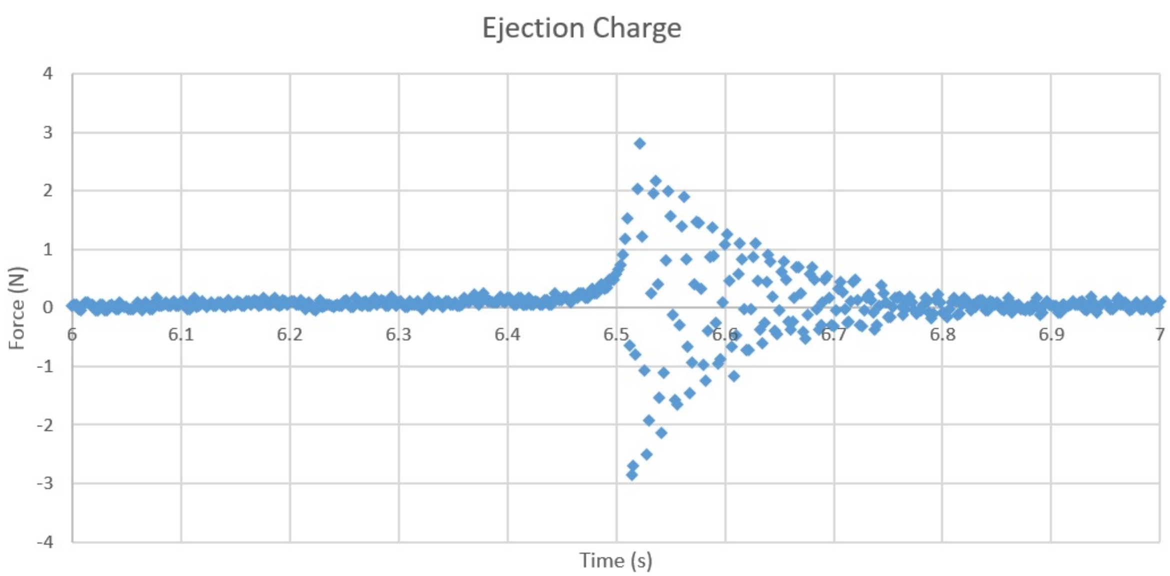

Figure 10: The ejection charge.

5.3.3 Total Impulse

The total impulse of the rocket is given by integrating the area under the force curve given in Figure 8. By integrating under the curve for the obvious duration of the test yields a total impulse of 8.6 Newton-seconds.

6 Conclusion

By carefully designing and constructing a thrust transducer, accurate thrust data for a rocket can be obtained using almost exclusively mechanical properties of aluminum and copper as well as some simple electrical circuits.